ACTIVATION OF VAPORIZATION CENTERS. II

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An approximate analytical expression is obtained for the solution of the system of equations describing the growth of vapor bubbles on a solid substrate in surface indentations.

This paper is a continuation of the author's earlier analysis [1]. The law of motion applicable to the surface of a bubble lying within an indentation on a rough solid substrate is determined by solving the system of equations (6) and (14)-(17) taken from [1]. In order to avoid certain mathematical difficulties (not of a fundamental nature) we shall assume that in Eq. (17) of [1] $\alpha = 0$. The temperature of the vapor in the bubble is determined by the expression $T_*(t) = T_0 + \Theta(0, t)$ derived from a solution of Eq. (6) on the basis of the integral method of Tolubinskii [2, 3]. The Green's function based on this method [2, 3] is given in [4].

The saturated vapor pressure is related to T_* by the equation

$$P_s(t) = aT_*^{\alpha} \exp\left(-\frac{\beta}{T_*}\right), \quad a = \exp u, \quad \alpha = \frac{\Delta C_p}{R}, \quad \beta = \frac{L}{R}, \tag{1}$$

derived from (16), while the vapor pressure in the bubble is

$$P_b(t) = A\,\frac{T_*(t)\,m(t)}{z^3(t)}, \quad A = \frac{3R}{\pi\Psi_1}\left(\frac{z_0}{r_0}\right)^2. \tag{2}$$

The law of mass variation of the vapor in the bubble which enters into Eq. (2) may be obtained together with the law of motion of the bubble's surface from the system of equations

$$\begin{gathered} m(t) = B\int_{t_0}^{t} f_1(t)\,z^2(t)\,dt - B\int_{t_0}^{t} f_2(t)\,\frac{m(t)}{z(t)}\,dt, \\ z\,\frac{d^2z}{dt^2} + q\left(\frac{dz}{dt}\right)^2 + \frac{n}{z}\,\frac{dz}{dt} = f_3(t)\,\frac{m(t)}{z^3} - \frac{D}{z} - F, \end{gathered} \tag{3}$$

in which we have introduced the notation

$$q = \frac{3}{2}, \quad n = \frac{10}{3}\nu, \quad D = \frac{2\sigma}{\rho_0}, \quad F = \frac{P}{\rho_0}, \quad A_0T_* = f_3(t),$$
$$A_0 = \frac{A}{\rho_0},$$

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$$B=\pi\eta\left(\frac{r_0}{z_0}\right)^2\left(\frac{M_0}{2\pi R}\right)^{1/2},\quad f_1(t)=aT_*^{\alpha-\frac{1}{2}}\exp\left(-\frac{\beta}{T_*}\right),$$

$$f_2(t)=AT_*^{1/2}.$$

An exact solution of the system (3) is hardly possible. Let us assume that in the equation for $m(t)z(t) = z_2+\gamma(t-t_0)$. The value of the coefficient γ may then be refined on the basis of the resultant solution for $z(t)$. To a first approximation γ may be determined from the equation $z_2+\gamma(t_1-t_0) = z_0+r_0$, in which the instant of time t_1 is found on the basis of the experimental law $z^*(t)$, using the equation $z^*(t_1) = z_0+r_0$. Another way of determining γ not depending on the results of the experiment will be indicated shortly. After this the solution of the equation

$$m(t)=\varphi(t)-B\int_{t_0}^{t}f(t)\,m(t)\,dt,\quad \varphi(t)=B\int_{t_0}^{t}f_1(t)\,[z_2+\gamma(t-t_0)]^2\,dt, \tag{4}$$

obtained by the method of Sokolov [5], has the following first-approximation form:

$$m(t)=\varphi(t)-\frac{B}{D_0(t-t_0)}\int_{t_0}^{t}\varphi(t')\,dt'\int_{t_0}^{t}f(t)\,dt\equiv\Phi(t), \tag{5}$$

where

$$D_0=1+\frac{1}{t-t_0}\int_{t_0}^{t}dt\int_{t_0}^{t}f(t')\,dt';\quad f(t)=\frac{f_2(t)}{z_2+\gamma(t-t_0)}.$$

If necessary higher approximations may also be obtained for $m(t)$ on the basis of [5].

We write the equation for $z(t)$ in the form

$$\frac{d^2z}{dt^2}+\frac{q}{z}\left(\frac{dz}{dt}\right)^2+\frac{n}{z^2}\,\frac{dz}{dt}+\frac{D}{z^2}+\frac{F}{z}=\Psi_0(t),$$

$$\Psi_0(t)=\frac{f_3(t)\,\Phi(t)}{[z_2+\gamma(t-t_0)]^4}. \tag{6}$$

The right-hand side allows for the fact that the vapor pressure in the bubble is variable in time and depends on the law governing the inflow of heat to the bubble and the motion of its boundary. We shall seek the solution of (6) in the form of a sum comprising the solution of the homogeneous equation with the corresponding initial conditions and a term allowing for the influence of the source $\Psi_0(t)$.

By substituting the variable $z' = dz/dt = p$ the homogeneous equation for z is reduced to an Abelian equation of the second kind. The substitution $u(z) = pz^q$ brings this to the form

$$uu'+nz^{-\frac{1}{2}}u+Dz+Fz^2=0. \tag{7}$$

We may simplify this equation by assuming that $z^{-1/2}\approx z_{av}^{-1/2} = [\frac{1}{2}(z_2+z_0+r_0)]^{-1/2}$, while $z^2\approx zz_{av}$. If we apply the parametrization method to the equation obtained in this way [6], we find that

$$\int\frac{dk}{(a_0+k)\,[M+(a_0+\gamma_0)\,k]}-\int\frac{dz}{Mz}=\text{const}, \tag{8}$$

$$a_0=nz_{av}^{-\frac{1}{2}},\quad k=u',\quad M=D+Fz_{av},\quad \gamma_0=\gamma z_{av}^{1/2}.$$

This leads to the relationship

$$\frac{dz}{dt}=\frac{C_1}{1-\frac{\mu}{M}}z^{1-q-\frac{\mu}{M}}-Nz^{1-q}\equiv E(z),\quad N=\frac{M}{a_0+\gamma_0},\quad \mu=M-a_0(a_0+\gamma_0) \tag{9}$$

and an approximation relation between z and t in the form

$$t+C_2=\frac{a_0}{C_1N}\left(\frac{1}{n_1}z^{n_1}+\frac{a_0}{C_1}\frac{1}{n_2}z^{n_2}\right),\quad n_1=\frac{5}{2}-\frac{a_0}{N},\quad n_2=\frac{7}{2}-\frac{2a_0}{N}. \tag{10}$$

The constants C_1 and C_2 are found from the initial conditions $t = t_0$, $z = z_2$, $dz/dt = E(t)|_{z=z_2} = 0$. After some calculations we find

$$C_1=a_0z_2^{1-\frac{a_0}{N}},\quad C_2=-t_0+\frac{a_0}{C_1N}\left(\frac{1}{n_1}z_2^{n_1}+\frac{a_0}{C_1}\frac{1}{n_2}z_2^{n_2}\right). \tag{11}$$

As a result of this the z(t) relationship for $\Psi_0(t) = 0$ follows from the equation

$$t-t_0=\frac{1}{Nz_2^{1-\frac{a_0}{N}}}\left[\frac{1}{n_1}(z^{n_1}-z_2^{n_1})+\frac{1}{z_2^{1-\frac{a_0}{N}}}\frac{1}{n_2}(z^{n_2}-z_2^{n_2})\right]. \tag{12}$$

The influence of the source $\Psi_0(t)$ may be taken into account by convoluting it [7] with the fundamental solution of the linearized equation (6). We write the latter approximately in the form

$$Lz\equiv\frac{d^2z}{dt^2}+g(t)\frac{dz}{dt}=\Psi_0(t)-\frac{D}{[z_2+\gamma(t-t_0)]^2}-\frac{F}{z_2+\gamma(t-t_0)}\equiv H(t),\quad g(t)=\frac{q\gamma}{z_2+\gamma(t-t_0)}+\frac{n}{[z_2+\gamma(t-t_0)]^2}. \tag{13}$$

Then we have $\Gamma(t-t') = L^{-1}\delta(t-t')$, where L^{-1} is an operator inverse to the original one. After carrying out the calculations

$$\Gamma(t-t')=\frac{\Theta(t-t')}{g(t)}\{1-\exp[-g(t)(t-t')]\},\quad \Theta(x)=\begin{cases}1, & x>0,\\ 0, & x<0.\end{cases} \tag{14}$$

Consequently, the approximate solution of Eq. (6) takes the form

$$z(t)=\int_{t_0}^{t}\Gamma(t-t')H(t')\,dt'+z_0(t), \tag{15}$$

where $z_0(t)$ is expression (12) solved for z. In the particular case of $a_0/N = 1$

$$z_0(t)=\left[\frac{3N}{4}(t-t_0)+z_2^{3/2}\right]^{2/3}. \tag{16}$$

As coefficient γ we may to a first approximation take

$$\gamma=\left.\frac{dz_0(t)}{dt}\right|_{z=z_{av}}=Nz_2^{\varepsilon}\left[z_{av}^{n_1-1}+\frac{1}{z_2^{\varepsilon}}z_{av}^{n_2-1}\right]^{-1},\quad \varepsilon=1-\frac{a_0}{N}. \tag{17}$$

The next approximation for z may be obtained after refining the coefficient γ on the basis of Eq. (15).

In an analogous way we may obtain an approximate solution for the more complicated system of equations (18)-(23) in [1]. The law of variation of the bubble radius r(t) in this system is qualitatively described by the solution (15). If we consider the process taking place in a single vaporization center, not from the instant at which heat is first conveyed to it as earlier, but from the instant at which the indentation is filled with the gas—vapor mixture, the analysis is slightly simplified. The high initial superheating of the liquid (for molten metals about 200°C in the absence of dissolved gases) is greatly reduced after the separation of the first bubbles because the indentation is then partly or completely filled with the gas—vapor mixture.

The system of equations proposed in [1] also (in a particular case) describes the growth of a vapor bubble in a large volume of superheated liquid at a temperature $T_0 > T_S$. In this case the problem is spherically symmetrical with respect to the center of the bubble, so that the boundary condition for the heat-conduction equation at x = 0 vanishes. In order to find T(r, t) in this case we may use the method of perturbations employed in [9] in order to analyze the radiative cooling of substances of arbitrary shape and variable volume. Here the Green's function of the second boundary problem of heat conduction in an unbounded liquid surrounding the bubble derived from [4] takes the form

$$\Gamma(r, r', t, t') = \frac{\exp\left[-\frac{r - r' - r_b(t)}{4a(t-t')}\right] - \exp\left[-\frac{r + r' - r_b(t)}{4a(t-t')}\right]}{8\pi r' \, [r - r_b(t)] \, [\pi a (t-t')]^{1/2}}. \tag{18}$$

Here a is the thermal diffusivity of the liquid; r' and t' are the radius of a single instantaneous spherical heat source and the instant of its appearance.

In analyzing the growth of a vapor bubble on a heated surface enveloped in a flow of liquid, the heat-conduction equation in the flow of liquid must be added to Eqs. (18)-(22) of [1]. In order to analyze the temperature pulsations in the solid wall under a unit center of vaporization, the proposed system of equations should be supplemented by the heat-conduction equation in the solid wall. The system of equations from [1] may be used to analyze cavitation processes in the superheated liquid. In this case we must add the diffusive mass flow arising as a result of gases dissolved in the liquid (see [10], for example) to the right-hand side of Eq. (20) taken from [1].

The foregoing time dependence of the radius of the growing bubble depends considerably on the physical properties of the liquid, its pressure, and the conditions underlying the flow of heat to the vapor bubble. It follows from Eqs. (12) and (15) that ν, σ, ρ_0 and P enter into the power index attached to z in a functional dependence t = F(z). The resultant z(t) law differs from the widely employed approximate law $z \sim \sqrt{t}$. From the solution of the system of equations we also derive the time dependence of the remaining parameters characterizing the growth of the bubble; the latter cannot be obtained from the models proposed by other authors.

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